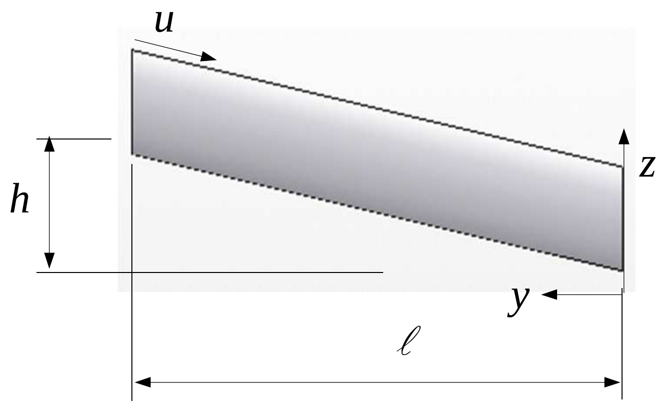
Rider Path Simulation for Waterslides

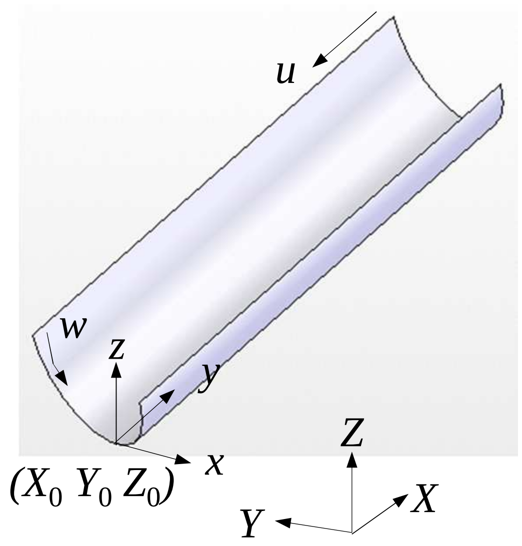
CAD Answers, LLC

Outline

- Geometric representation of the flume sections
- Mathematical formulation
- Numerical computation
- Example slide

Equations for Straight Section





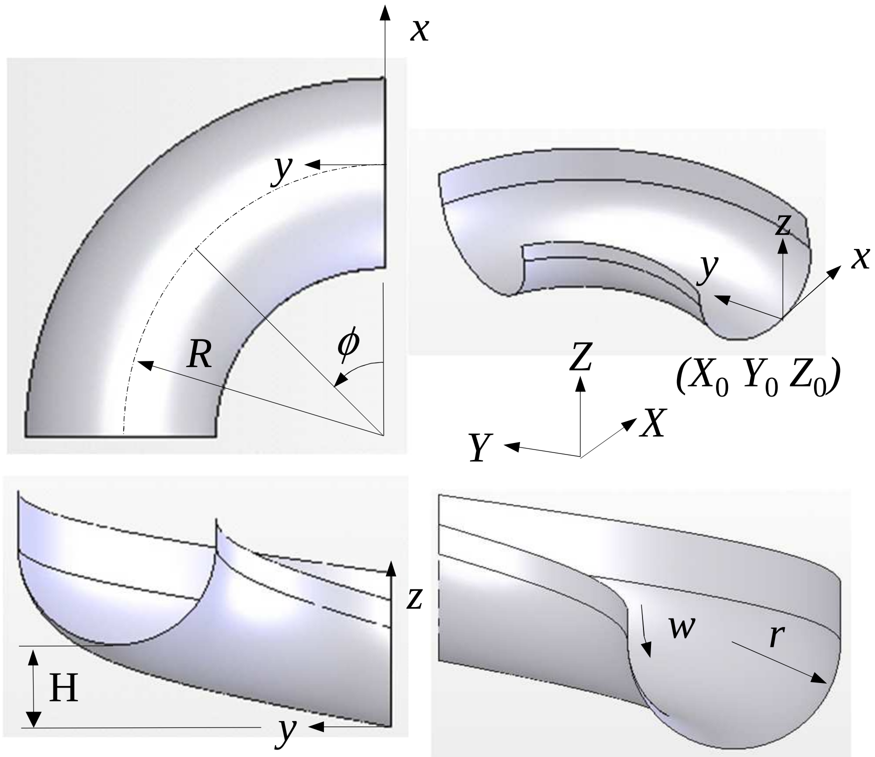
$$\mathbf{X}(u,w) = \mathbf{X}_0 + \mathbf{T}(\theta)\,\mathbf{x}(u,w)$$

$$= \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(u,w) \\ y(u,w) \\ z(u,w) \end{bmatrix}$$

$$\begin{bmatrix} x(u,w) \\ y(u,w) \\ z(u,w) \end{bmatrix} = \begin{bmatrix} -r\cos(w\pi) \\ \ell(1-u) \\ h(1-u) + r(1-\sin(w\pi)) \end{bmatrix}$$

$$(u,w) \in [0,1]$$

Equations for Elbow Section



CCW Elbow

Wall:

$$\begin{bmatrix} x(u,w) \\ y(u,w) \\ z(u,w) \end{bmatrix} = \begin{bmatrix} \cos u\phi\,(-R \pm r) \\ r(1-\sin w\pi) + H(1-u) \\ \sin u\phi(-R \pm r) + R \end{bmatrix}$$

CCW Elbow :

$$\mathbf{X}(u,w) = \mathbf{X}_0 + \mathbf{T}(\theta)\,\mathbf{x}(u,w)$$

$$= \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(u,w) \\ y(u,w) \\ z(u,w) \end{bmatrix}$$

$$\begin{bmatrix} x(u,w) \\ y(u,w) \\ z(u,w) \end{bmatrix} = \begin{bmatrix} (R - r\cos(w\pi))\cos((1-u)\phi) - R \\ (R - r\cos(w\pi))\sin((1-u)\phi) \\ r(1-\sin(w\pi)) + H(1-u) \end{bmatrix}$$

$(u,w) \in [0,1]$

CW Elbow :

$$\begin{bmatrix} x(u,w) \\ y(u,w) \\ z(u,w) \end{bmatrix} = \begin{bmatrix} (-R - r\cos(w\pi))\cos((1-u)\phi) + R \\ (-R - r\cos(w\pi))\sin((1-u)\phi) \\ r(1-\sin(w\pi)) + H(1-u) \end{bmatrix}$$

$(u,w) \in [0,1]$

Motion Analysis Theory

- Lagrange's Equations of Motion

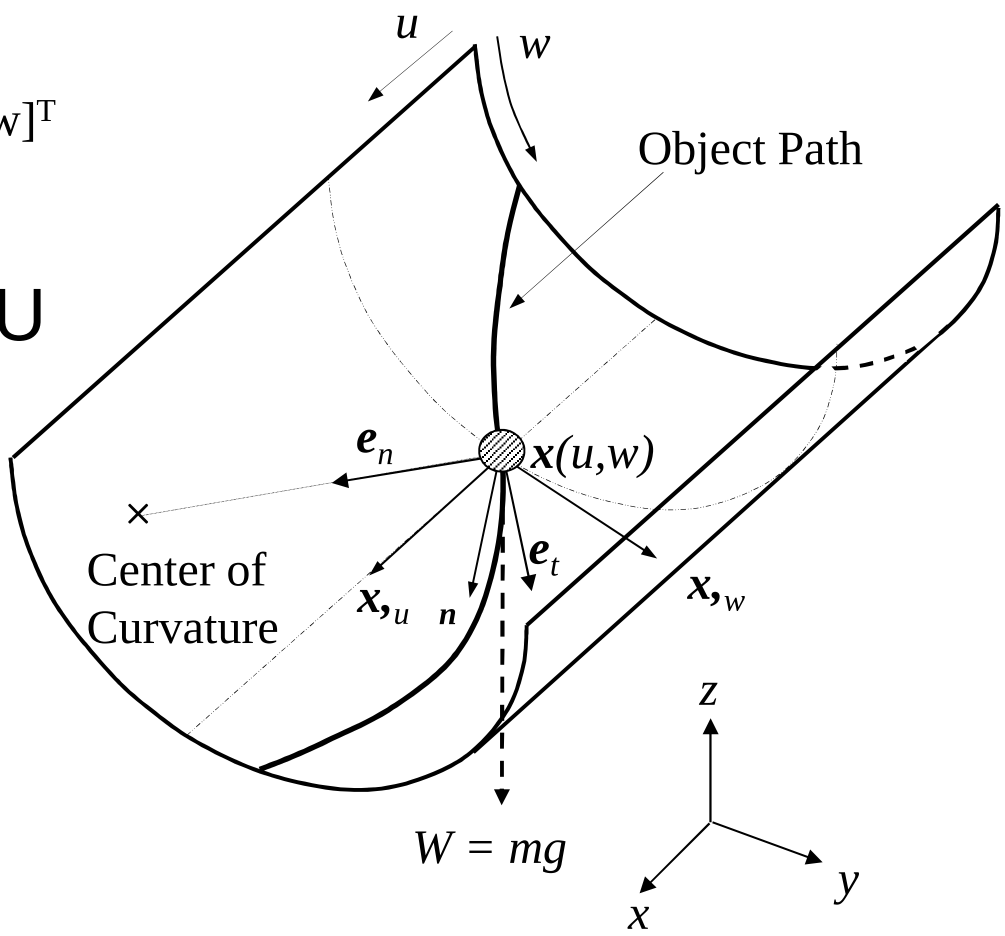
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial L}{\partial \mathbf{q}} = \mathbf{Q} \qquad \mathbf{q} = [u,w]^T$$

- Lagrangian: L = T–U
  - T: kinetic energy

$$T = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

  - U: potential energy

$$U = mgz(u,w)$$



Motion Analysis Theory

- Friction Forces

$$Q_u = \mu(mgn_z + m\mathbf{a}_n \cdot \mathbf{n})(-\mathbf{e}_t \cdot \mathbf{x},_u)$$

$$Q_w = \mu(mgn_z + m\mathbf{a}_n \cdot \mathbf{n})(-\mathbf{e}_t \cdot \mathbf{x},_w)$$

- Ordinary Differential Equations

$$k_0 \ddot{u} = k_1 \dot{u}^2 + k_2 \dot{w}^2 + k_3 \dot{u}\dot{w} + k_4$$

$$k_0 \ddot{w} = k_5 \dot{u}^2 + k_6 \dot{w}^2 + k_7 \dot{u}\dot{w} + k_8$$

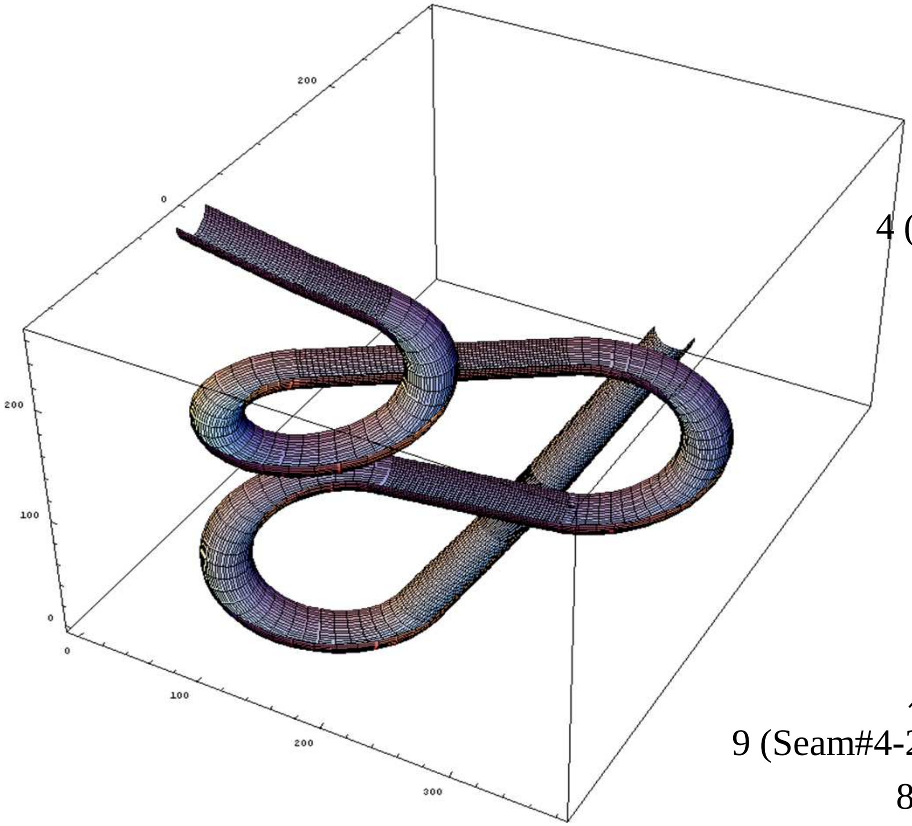
- Initial Conditions:

$$u(0) = u^0, w(0) = w^0, \dot{u}(0) = \dot{u}^0, \dot{w}(0) = \dot{w}^0$$

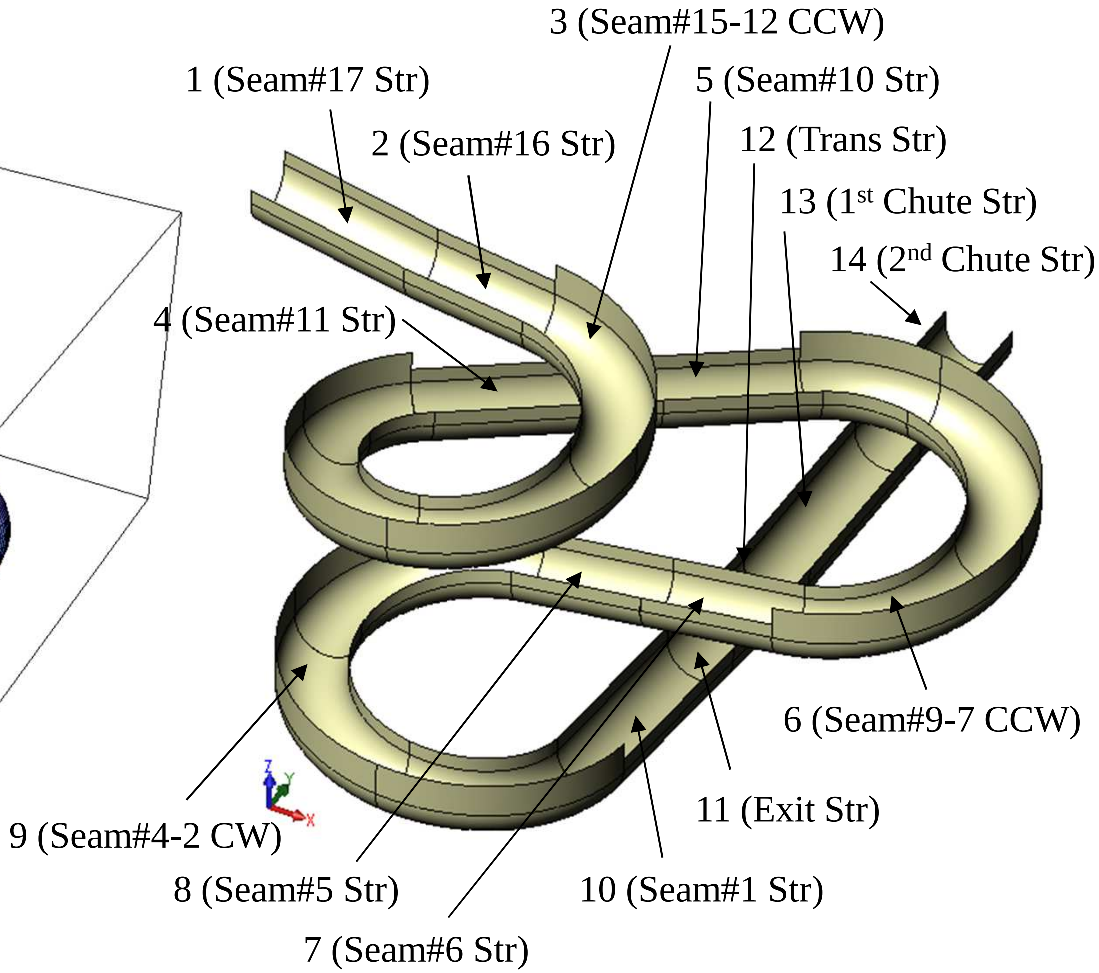
Numerical Computation

- Solving Equations of Motion
  - Mathematica's NDSolve function
  - Adams Predictor-Corrector and Gear methods for solving the ordinary differential equations numerically

Example Slide

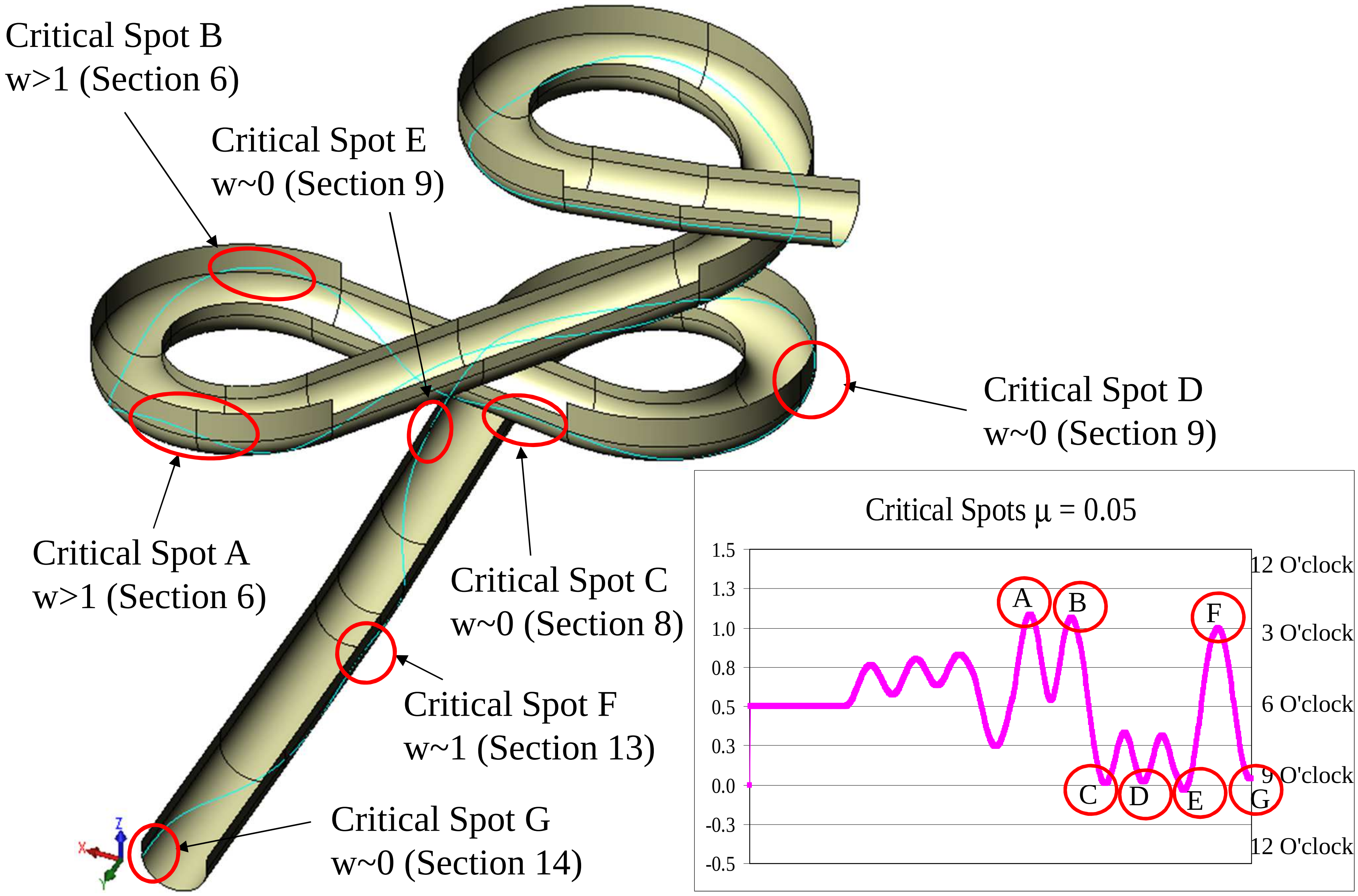


Mathematica



SolidWorks

Sample Case 3: μ = 0.05



Sample Case 3 (cont’d)

- Total time: 11.28 seconds (Sections 1-11)
- w value is beyond (or very close to the limits of) the regular 3-9 O’clock range ($0.0 < w < 1.0$) at 7 locations
- Maximum w value is 1.1 and minimum is –0.04
- Average velocity: 11.00 MPH
- Max velocity: 15.08 MPH
- Max acceleration: 2.57 g

Summary

- Reasonably accurate simulations have been obtained for the slide example, based on the Lagrange's equations of motion for the rider with concentrated mass.
- The simulation results depend on the rider's initial position and velocity, and other factors such as friction. The results simulated assumed specific conditions.
- Case 3 (μ = 0.05) reveal that the rider slides very close to or over the edge of the flume sections at multiple locations. The simulated riding time and average velocity match well with measured data.
- Even though the side walls (8'' and 18" tall) provide some protection, the results raise flags in riders' safety, especially for experienced and aggressive riders.

Disclaimer

- Simulation results do not answer all the questions regarding safety of the slide. However, they do provide accurate predictions on the positions, velocities, and accelerations of the riding object.
- The safety issues must be further ensured by adequate tests carried out on the slide once it is built as well as regular safety inspections performed by experts. These are beyond the scope of the simulation task summarized in this document.